# Possibilistic preference elicitation by Minimax regret

## GT R & A

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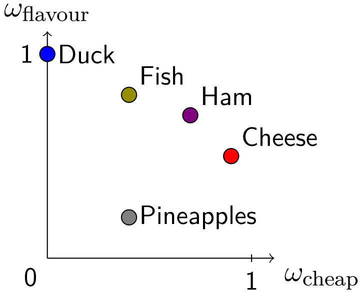
# Plan

# Example: multiple-criteria decision

- Questions: how can I find my favourite pizza?

| | Flavour | Cheap |
|---|---|---|
| Cheese | 5 | 9 |
| Duck | 10 | 0 |
| Fish | 8 | 4 |
| Ham | 7 | 7 |
| Pineapples | 2 | 4 |




- Supposition: agent's preferences = aggregation function.
- Here: $f_\omega(\text{pizza}) = 0.6\ \text{flavour} + 0.4\ \text{cheap}$.

# Example: multiple-criteria decision

- Questions: how can I find my favourite pizza?

| | Flavour | Cheap | $f_\omega$ |
|---|---|---|---|
| Cheese | 5 | 9 | 6.6 |
| Duck | 10 | 0 | 6 |
| Fish | 8 | 4 | 6.4 |
| Ham | 7 | 7 | 7 |
| ~~Pineapples~~ | ~~2~~ | ~~4~~ | ~~2.8~~ |

- Supposition: agent's preferences = aggregation function.
- Here: $f_\omega(\text{pizza}) = 0.6 \text{ flavour} + 0.4 \text{ cheap}$.
- Best: ham with a score of 7. Pineapples always dominated.

# Incremental elicitation

- Problem: in practice, the parameters $\omega$ are unknown.
- An expert chooses a parametric family of aggregate functions of criteria $f_\omega$ (weighted sum, OWA...) describing the preferences.
- The expert searches the parameters $\omega \in \Omega$ by eliciting the preferences of an agent through explicit questions (pairwise preferences).
- An interesting method is the incremental elicitation: questions depend on previous answers [1].

# Robust approach

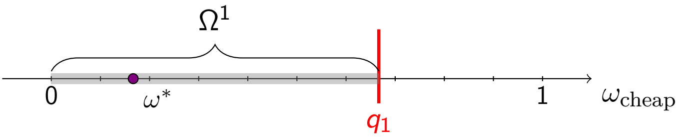
- One main approach for incremental elicitation is the robust optimisation (using Minimax regret) [3, 2].
- The idea: finding the subset of $\Omega$ satisfying all constraints.
- Pro: worst case performance guarantees.
- Con: strong hypotheses supposing no errors in the answers (oracle) and in the choice of $f_\omega \Rightarrow$ no management of uncertainty.

# Elicitation sequence and CSS

- At each step $k$, a pair $q_k = (x_k, y_k)$ is compared by the user.
- $x_k$ is the alternative which optimise a criterion (Minimax regret).
- Current Solution Strategy (CSS) [4] is a good heuristic to get $y_k$. $y_k$ is the worst opponent of $x_k$.
- The user picks either $x_k \succeq y_k$ or $x_k \preceq y_k$.
- The set of possible models $\Omega$ is updated.

# Process illustration: finding $\omega^*$

- The agent answers correctly three questions $q_1$, $q_2$ and $q_3$ by comparing each time two alternatives.
- Each answers refines the set of possible models such that $\omega^* \in \Omega^3 \subset \Omega^2 \subset \Omega^1 \subset \Omega$.



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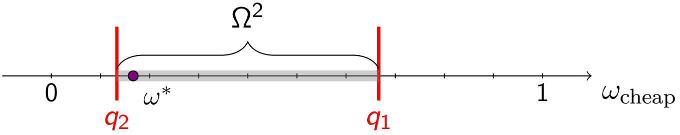

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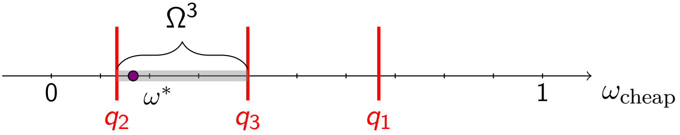
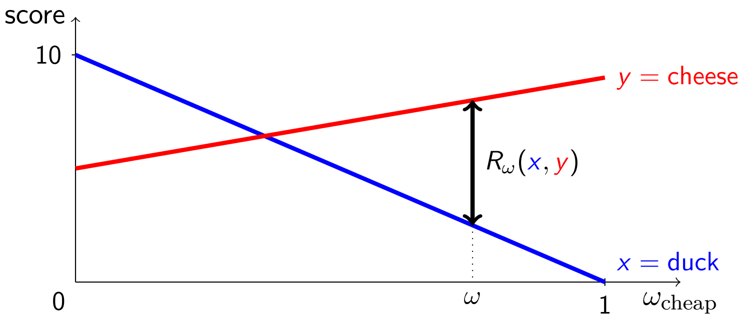


# Plan

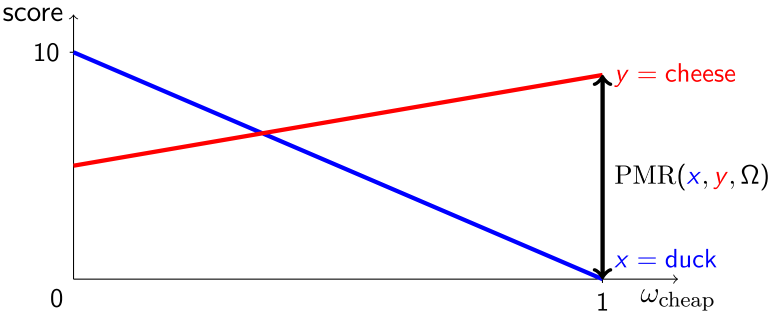
# Incremental elicitation with Minimax **regret**

Regret of taking $x$ instead of $y$ for a model $\omega \to \mathrm{R}_\omega(x, y) = f_\omega(y) - f_\omega(x)$.



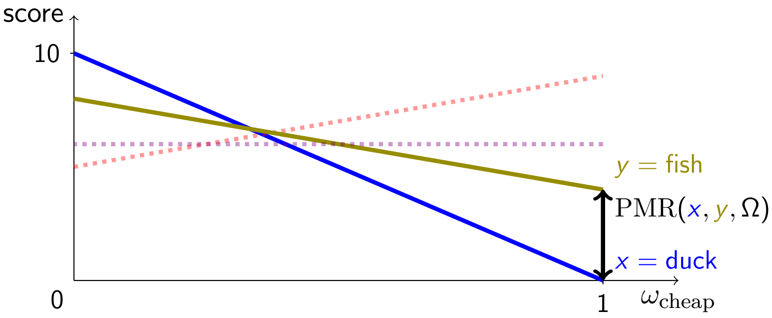
# Incremental elicitation with Minimax regret

Maximum regret of a pair $(x, y) \rightarrow \mathrm{PMR}(x, y, \Omega') = \max_{\omega \in \Omega'} \mathrm{R}_\omega(x, y)$.



# Incremental elicitation with Minimax regret

Maximum regret of $x \to \mathrm{MR}(x, \Omega') = \max_{y \in \mathbb{X}} \mathrm{PMR}(x, y, \Omega')$.



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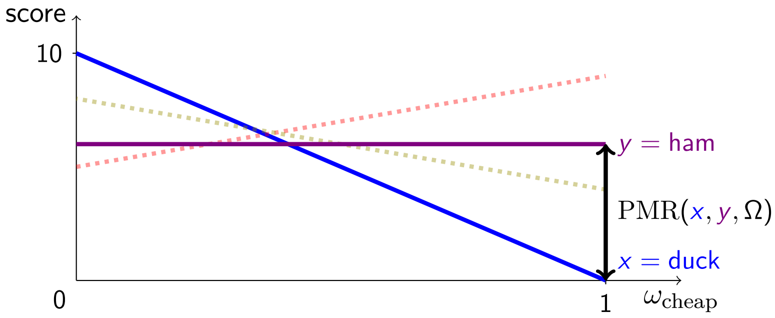

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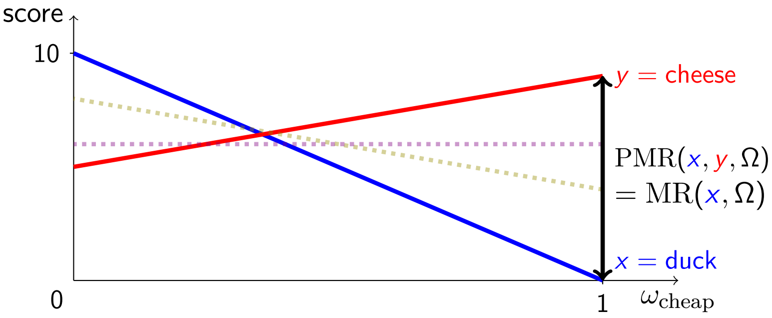
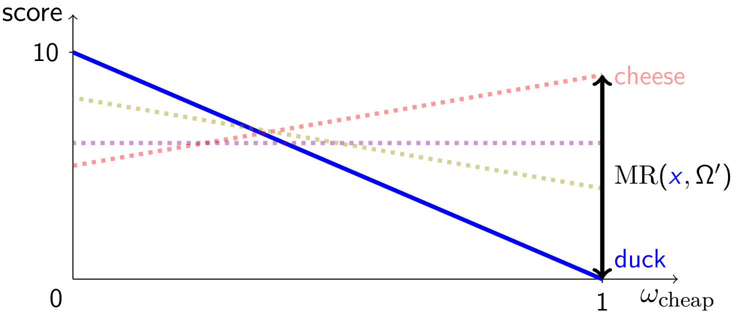


# Incremental elicitation with **Minimax regret**

Minimax regret $\rightarrow \mathrm{mMR}(\Omega') = \mathsf{min}_{x \in \mathbb{X}} \mathrm{MR}(x, \Omega')$.



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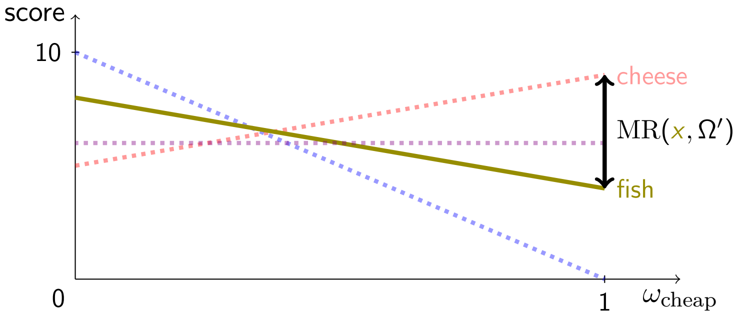

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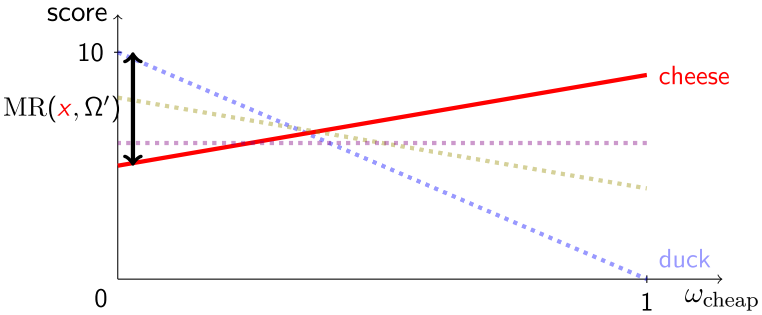

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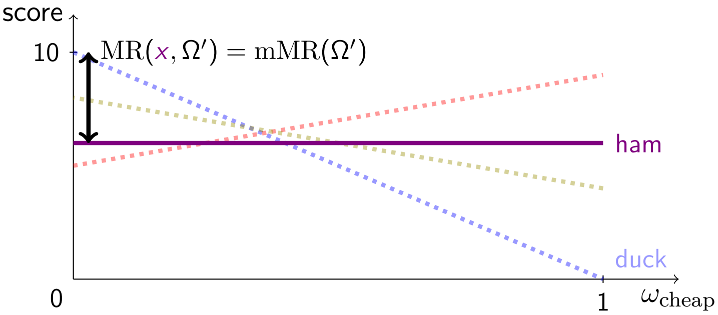
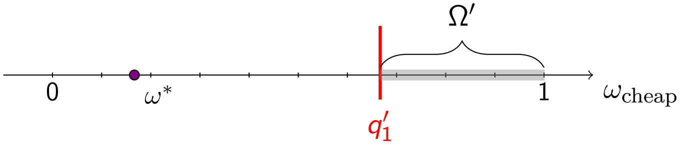


# Plan

# What happens in case of error?

- Let us suppose the agent gives a wrong answer to the question $q'_1$, then the optimal model $\omega^*$ is not part of $\Omega'$:




- Further questions will refine $\Omega'$, thus never returning to the optimal model.

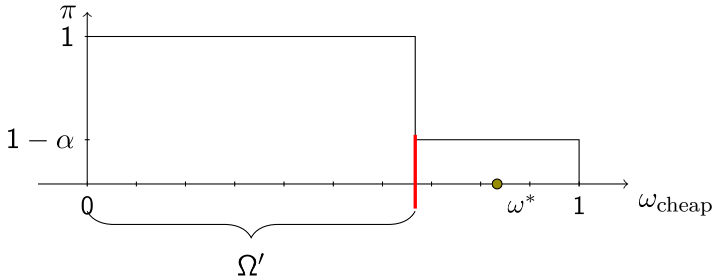
# Our solution with a possibilistic approach

- Objective: keeping the strong guarantees of the robust approach, while managing uncertainty arising from errors.
- We propose to extend robust approach with the possibility theory.
- Possibility theory: alternative to probabilities, generalising set theory, linked to fuzzy sets.
- Pros: managing errors and low computation complexity (less costly than when belief functions are used [5]).

# Possibilistic extension

- The agent gives a confidence degree $\alpha \in [0, 1]$ with each answer. 1 indicates certainty, 0 total uncertainty.
- This approach is an extension of the robust approach, with weighted version of PMR, MR and mMR ($\alpha = 1 \Leftrightarrow$ robust approach).
- It is possible to detect an inconsistency easily.

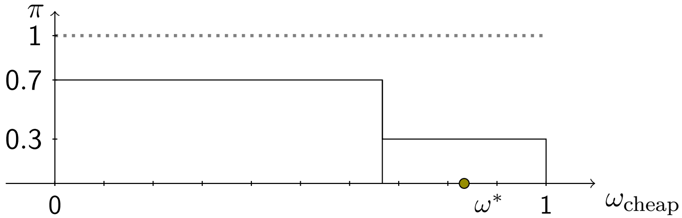
# Illustration of possibilistic elicitation




- Even if the agent gives a wrong answer, it's possible to return to $\Omega \setminus \Omega'$ and thus to find $\omega^*$.

# Illustration of inconsistency

- Let us suppose the agent gave some wrong answers. We may have such a situation:



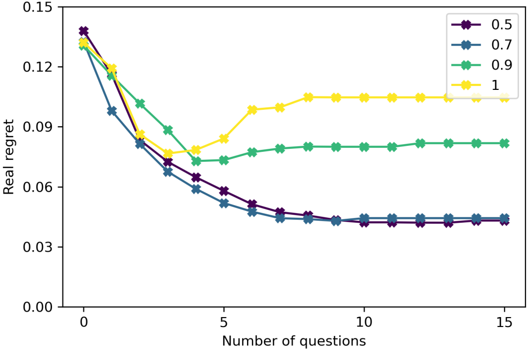

- Given the maximum of the function $\pi$ is inferior to 1, it means an inconsistency has been detected. Here $K = 0.3$.

# Plan

# Experimental protocol

- Two different experiments to test our approach:
  - Experiment 1: the agent gives some errors (30% of answers are random).
  - Experiment 2: wrong function (supposed: weighted sum).
- 200 numerical simulations per experiment:
  - 35 alternatives with 4 criteria generated randomly (uniform).
  - Aggregating functions generated randomly (Dirichlet).
- Stop after 15 questions, if the regret no longer decreases, or when an inconsistency is detected.
- Results averaged over the 200 simulations.

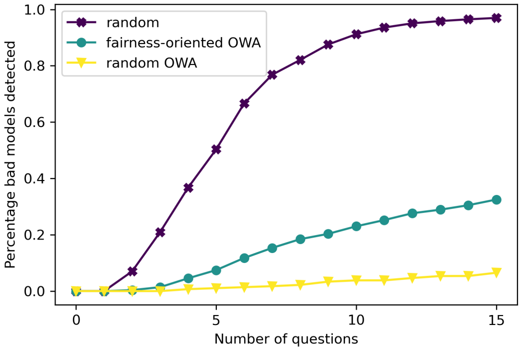
# Experiment 1: performances of strategies




- Robust ($\alpha = 1$): no detection $\Rightarrow$ move away from the right model.
- Possibilistic: detection of inconsistency $\Rightarrow$ stops before being too away from the right model.

**Possibilistic approach avoids being stuck on a bad region**

# Experiment 2: detecting wrong $f_\omega$, $\alpha = 0.7$




- Our approach is able to detect the inconsistency more or less quickly depending on the model.
- The more distant a model is, the quicker the detection is.

**Possibilistic appoach useful to detect model misspecification**

# Conclusion

- We proposed a possibilistic method, with strong performance guarantees, while managing uncertainty.
- We can detect inconsistency from wrong answers given by the agent, or from a wrong assumption of the model by the expert.
- Future works:
  - Repair the observed inconsistency:
    - Delete some pieces of information $\Rightarrow$ find answers that are coherent together (maximal coherent subset)?
    - Pick another model $\Rightarrow$ easy in theory, but only if the user is coherent.
  - Extend the approach to non numerical structures, lexicographic ones for example $\Rightarrow$ how to represent the regret?

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